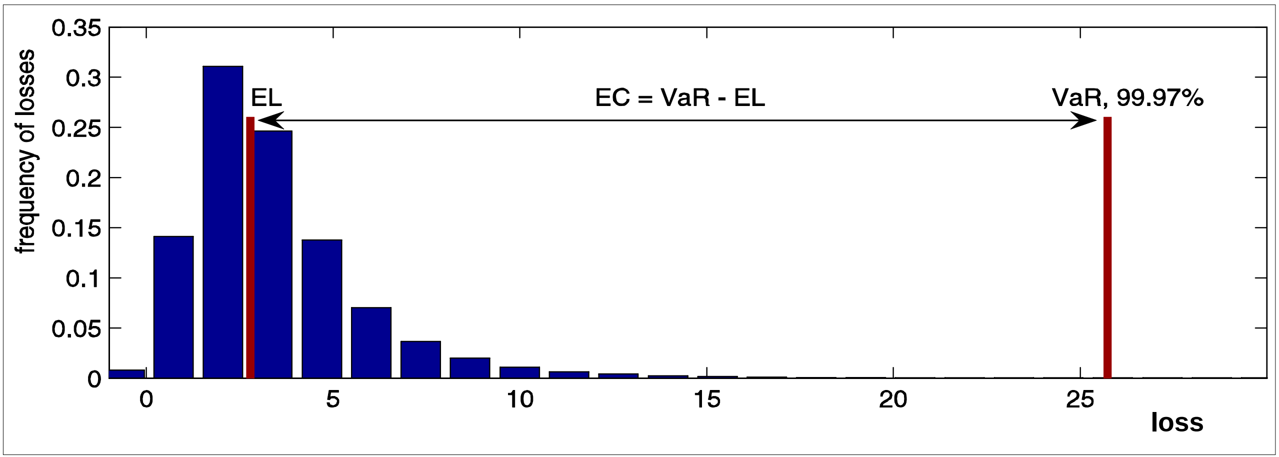
Dependency modelling in credit risk

Banks as well as other companies need capital to withstand potential future losses. Banks assess the minimum size of their capital base using various methods, including internally developed models.

One common approach to estimating the capital needed to cover credit losses within a bank's portfolio of loans is to simulate the future state of the portfolio. The outcome of the simulation is a loss distribution which enables the bank to calculate the recommended minimum capital base, taking into account the desired external rating[1] and the risk appetite of the bank.

Figure 1 below shows a schematically drawn loss distribution. *Expected loss*, EL, is the average value of losses 'observed' in the simulations, and *economic capital*, EC, is the 99.97th percentile loss minus EL[2]. If a suitable spread is added to the interest rate of the loans, the revenue from the spread covers EL. EC, on the other hand, is the recommended minimum capital base and is supposed to cover any losses exceeding expected loss. With EC calculated at a 99.97% confidence level the bank should be able to cover its losses within the next year in 9,997 out of every 10,000 cases.

Figure 1 Loss distribution (schematic)



An accurate estimate of EC requires an accurate description of the tail of the loss distribution. One of the main contributors to this tail is the *correlation* between obligors, i.e. the extent to which obligors tend to default on their loans simultaneously. The more correlated the obligors, the more extended the tail, the higher becomes EC. Thus, modelling correlations (or *dependency modelling*) is paramount to credit risk modelling and seems to be one of the great challenges facing most banks.

Correlations are commonly modelled using a so-called *factor model*. That is, the financial health of obligor i depends on a set of *systematic risk drivers*, $\mathbf{X} = (X_1, ..., X_M)$, and an *idiosyncratic term*, ε_i :

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[1] Rating agencies like Moody's, Standard & Poors and Fitch Rating assign ratings to banks on a regular basis. The resulting external rating has a significant impact on the bank's cost of lending money from other banks.

[2] Other percentiles than the 99.97th can be used here. In general, the higher the desired external rating, the higher the percentile.

$$r_i = R_i \left[\sum_{j=1}^{M} \alpha_{ij} X_j \right] + \sqrt{1 - R_i^2}\, \varepsilon_i \qquad (1)$$

In the simulations, obligor *i* defaults if the value of r_i falls below a certain threshold.

The chosen systematic drivers are typically equity indices. For instance, an ordinary Danish retail customer could be mapped to MSCI Denmark which would then reflect the macroeconomic environment surrounding the customer, whereas the idiosyncratic term would reflect the unique circumstances of this particular customer, i.e. things that are not related to the state of the economy.

In the simulations the X_j's can be taken to be normally distributed with variances and correlations derived from recent market data. For instance, one could use 5 years of monthly observations to derive the variances of the equity indices and the correlations between them. The ε_i's are taken to be independent, standard normally distributed, independent of the X_j's.

In the set-up described in the above, the correlation between obligors is determined by

1. the correlations between the drivers,
2. the sensitivity of each obligor to the drivers (R_i and the α_{ij}'s).

As for item 1, the estimated correlation between the drivers and the correlation dynamics depend on the chosen data window (the longer the window, the more stable the parameters) and the frequency of observations (daily, monthly, etc.) When measuring capital on a one-year horizon, one wants the correlations to reflect the immediate future as well as possible.

We would like the working group to look into the following problems:

a) The factor model described in the above may not be the optimal way of modelling dependency between obligors. Also, the intuitive link between equity indices and customer default is not as strong as the link between e.g. macro variables and default. Are there alternative ways, alternative data etc. which would provide a better and more intuitive description?

b) If one uses the factor model described above, then what can be done to ensure correlation estimates which

   i. are sufficiently dynamic to capture changes in the market (e.g. the financial crisis) but still not too volatile?
   ii. contain a forward-looking element as well?

c) In the factor model described above, how can one determine the R_i's and the α_{ij}'s which provide the most accurate description of correlations? And how can the parameters be validated?