Gas portfolio optimisation under uncertainty

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1 Introduction

As a consequence of the liberalisation of the Energy markets in Europe, gas producers and wholesale participants face a new class of optimisation problems. Gas is not only purchased on long term contracts, but is also traded on a growing and increasing important spot market. Thus, long term contracts is not only used for securing a demand of gas in a monopolic market, but is actually also used for commercial spot trading. The same considerations apply for a storage, as not being only used for secure the demand in winter, but may also be used for commecial trading.
However, the spot markets around in Europe are still not perfect in an economic sense, and also there is several market areas for which, it is not possible to fully supply the demand by a spot market (Denmark/Sweden is such an example).

This situation gives rise to some challenging matemathical programs, which the gas industry is faced with every day. We will here define a problem, which is a simplification compared to real problems. However, it is our hope, if we can find a solution to this problem, and the methodology is scaleable, we may extend the methods to be used for real problems.

2 Problem description

We would like to find a supply plan for day t for the situation:

1. The gas supplier has one complex gas supply contract (see below for more details).
2. Access to one storage.
3. Obligations to deliver gas to a retail market RC.
4. Limited access to sell gas on a HUB (market place).

The physical network is shown below. The supplier purchases gas on a so called complex or long-term contract. Through Transmission System Operator 2 (TSO2) it is possible to flow a limited amount of the purchased gas to the HUB for selling it to a market price. Through Transmission System Operator 1 (TSO) the supplier may either source the retail market RC, or inject it into the storage. The storage may also be used for sourcing the retail market.

The characteristica of the purchase contract, storage, retail market and HUB market will be given below.

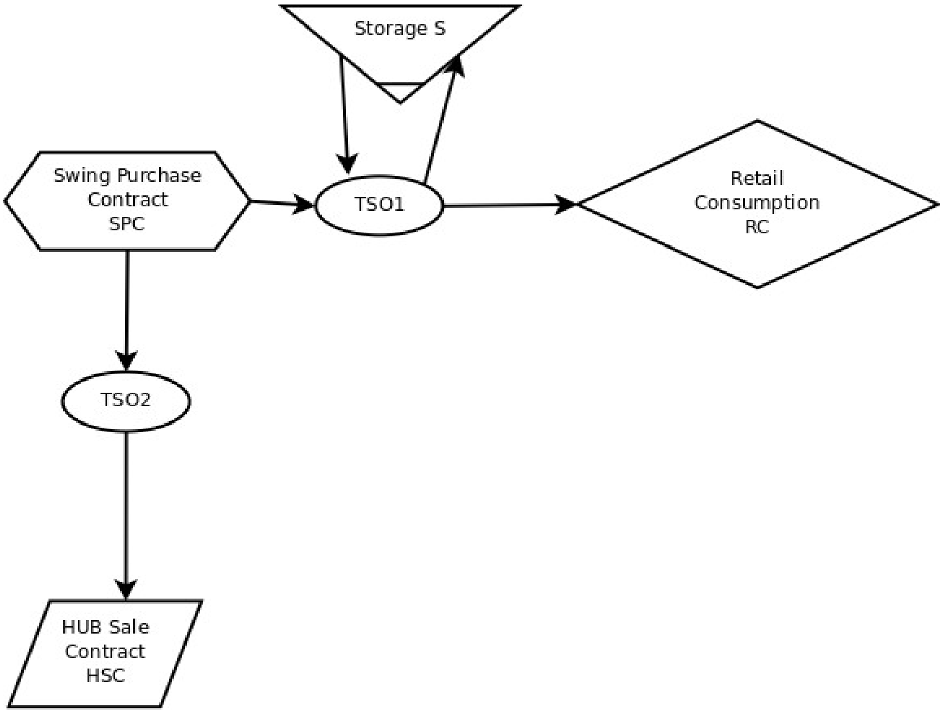

Illustration 1: Physical network

3 Retail market RC:

The consumption on the retail market is in this simplified version only dependent on the temperature. One simple approach for the relationships between consumption Q_t^{RC} on day *t* and temperature is to assume the consumption is normally distributed given temperature,

$$p(Q_t^{RC}|T)=N(-\mu\cdot T+\gamma,\sigma^2) \quad (1)$$

The temperature may also be assumed to follow a normal distribution given the day t in the year as in equation 2.

$$p(T|t)=N(C-\alpha\cdot\cos(\beta\cdot(t-t_0)),\epsilon^2) \quad (2)$$

t_0 is a reference day.

We are obliged to deliver the demanded quantum of gas. If we do not meet the demand, we can define the penalty cost as

$$C_t^{RC}=f(Q_t^{UD})\cdot(C_0^{RC}+P_t^{UD}\cdot Q_t^{UD}) \quad (3)$$

Where the undelivered quantity Q_t^{UD} is defined as

$$Q_t^{UD}=max(0,Q_t^{RC}+Q_t^{INJ}-Q_t^{SPC,1}-Q_t^{WD}) \quad (4)$$

and $f(x)$ is 1 if $x>0$, otherwise 0.

4 Swing Purchase contract:

For the supply contract we have that the purchase Q_t^{SPC} on the contract on day t may be between D^{min} and D^{max}. For a whole year (gas year) the total purchased amount of gas must be between Y^{min} and Y^{max}. To summarise,

$$\begin{aligned} D^{min} &\leq Q_t^{SPC} \leq D^{max} \\ Y^{min} &\leq \sum_{t=1}^{T_N} Q_t^{SPC} \leq Y^{max} \end{aligned} \quad (5)$$

As mentioned the purchased gas may be sold at the hub on the market area TSO2 or transported to the market area TSO1:

$$Q_t^{SPC}=Q_t^{SPC,1}+Q_t^{SPC,2} \quad (6)$$

$Q_t^{SPC,1}$ is the quantity going into market area 1, and $Q_t^{SPC,2}$ is the quantity going into market area 2.
Contract price is an indexed Oil price with 3 month time lag and 6 month reference period and is constant for the month. Thus, the price for month m is

$$P_m^{SPC}=P_0+\alpha\cdot\frac{1}{6}\cdot\sum_{i=(m-9)}^{i=m-4} P_i^{oil} \quad (7)$$

If we consider the gas price for April, the average of oil prices P_i^{oil} is for the months July to December the previous year.

You may assume here the index oil price follows an stochastic Ornstein-Uhlenbeck process. But in general we are looking for a solution, where we do not need to make any assumptions about the distribution of the oil price.

5 Storage Contract:

We assume here the supplier has booked the storage with the properties,

- The daily injection to storage Q_t^{INJ} must be less than injection capacity C^{INJ}.
- The daily withdrawal from storage Q_t^{WD} must be less than withdrawal capacity C^{WD}.
- The physical volume on the storage at the end of day t V_t must be less than the booked storage capacity V^{book}

Furthermore we have balance restriction:

$$V_t = V_{t-1} + Q_t^{INJ} - Q_t^{WD} \tag{8}$$

For the opening balance at the beginning of the year and closing balance at the end of year of the storage, we can imagine 2 different cases:

1. The opening and closing balance must equal each other, implying over time we have a static system.
2. It is possible to valuate the gas storage volumes, and the volumes will be part of the optimisation problem.

But we would like to discuss the boundary conditions for the storage at the workshop.

6 HUB sale:

It is possible to sell at the hub. To simplify the problem assume, the HUB price P_t^{HSC} follows a modified Ornstein-Uhlenbeck process, in which the mean value is seasonal dependent. But for a general solution we would prefer a methodology in which we do not have to make any assumptions about the distribution of the market price.

The quantity sold at the HUB is given as

$$Q_t^{HSC} = Q_t^{SPC,2} \tag{9}$$

and the quantity of gas, which can flow to the HUB is limited by a maximum capacity in the pipeline

$$Q_t^{SPC,2} \leq F^{max} \tag{10}$$

7 Problem:

For day t, how much gas shall we buy on the purchase contract, how much shall we sell at the HUB, and how should we utilise the storage. That is, we want to find a production plan for day t, so we maximize the expectation:

$$E\left(\sum_{t}^{t_N}\left(P_t^{HSC}\cdot Q_t^{HSC}-P_t^{SPC}\cdot Q_t^{SPC}-C_t^{RC}\right)\right) \tag{11}$$

subject to the constraints and conditions mentioned above.