# 2017 ESGI –PhD Course Abstracts:

## Selected Topics in Optimization

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Abstract:

**Stochastic Differential Equations: Theory, Applications, and Computations**

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Abstract:

Stochastic differential equations (SDE's) are used to model dynamic systems that evolve unpredictably in time, and where we explicitly want to address this unpredictability in the model. The motivations are typically to characterize the unpredictable fluctuations, to do statistics for stochastic processes, or to control systems in presence of uncertainty. SDE's can be seen as ordinary differential equations, where we add a stochastic noise term which continuously perturbs the state. While much of the theory of SDE's builds on the theory of ODE's, the solutions of SDE's are not differentiable, and this entails a different calculus, viz. the Itô calculus. Here, we will state the central results in this calculus, notably the chain rule, and demonstrate how it can be used to e.g. change coordinate systems and verify solutions of SDE's.

Solutions of SDE's can rarely be found analytically, so numerical methods are important for applications. Here, we will describe how to perform Monte Carlo simulations of solutions to SDE's, using the Euler scheme or extensions of it. Beyond Monte Carlo simulations, analysis of the statistical properties of the solutions often reduce to partial differential equations of advection-diffusion type. This is true because the solutions of SDE's can be viewed as Markov processes, where the transition probabilities are governed by advection-diffusion equations. We will show a couple of these links between SDE's and PDE's, and demonstrate numerical methods for analysis of the resulting PDE's. In addition, since estimation of states and parameters is a key area of application, we will show principles and computational approaches to these estimation problems.

The course consists of theoretical lectures, demonstrations of applications and of software, exercises using pen and paper and computer exercises using R, Matlab and/or python.